Grade 7/8 Math Circles

Week of 13th November

Types of Numbers - Problem Set

1. What sets do the following numbers belong to? $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \overline{\mathbb{Q}}, \mathbb{R}, \mathbb{I}$, or $\mathbb{C}$
   (a) $3 + 3i$
   (b) π
   (c) 0
   (d) $-\frac{17}{\sqrt{2}}$
   (e) $\sqrt{5}i$
2. Determine which of the following numbers are elements of $\bar{\mathbb{Q}}$. That is, which of the following numbers are irrational?
   (a) $\sqrt{16}$
   (b) $\frac{5}{6}$
   (c) 0
   (d) $3.\overline{99}$
   (e) $\sqrt{5}$
3. Evaluate the following expressions.
   (a) $(2 + 3i) + (3 - \frac{1}{2}i)$
   (b) $(2 - 4i) - (3 + 4i)$
   (c) $(1 - 2i) \cdot (2 + 2i)$
   (d) $(3 - 4i) + ((1 - 3i) \cdot (1 + 2i))$
4. Evaluate the following expressions.
   (a) $\frac{1+2i}{2-i}$
   (b) $|4 + 7i|$
   (c) $\frac{5-4i}{3+4i}$
   (d) $\frac{3-4i}{5+12i}$
5. Answer the following true/false questions. If the statement is false, give a counterexample.
   (a) The product of two irrational numbers is always irrational.
   (b) The product of two integers always an integer.

(c) The product of two complex numbers is always complex.

(d) The product of two natural numbers is always a real number.

6. What values of x satisfy the following equations? State the type of number x is.

   (a) $x^2 + 1 = 0$

   (b) $x^2 = -36$

   (c) $x^2 + 2 = 0$

   (d) $x^2 + 1 = \frac{1}{2}$

7. Determine whether or not the following expressions yield a rational, or irrational number. Given $x = 5$, and $y = 4$.

   (a) $\sqrt{x + y}$

   (b) $\sqrt{x - y}$

   (c) $\sqrt{x \cdot y}$

   (d) $\sqrt{x/y}$

8. Evaluate the following set expressions.

   (a) $A \cap \overline{A}$

   (b) $\mathbb{N} \cup \mathbb{I}$

   (c) $\mathbb{R} \cap \mathbb{C}$

   (d) $\{a, b, c, d, e, ...\} \cap \{a, e, i, o, y, u\}$